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BOUNDARY LAYER FLOW AND HEAT TRANSEFER OF DUSTY FLUID OVER A STRETCHING SHEET WITH HEAT SOURCE/SINK

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ABSTRACT

In this paper, we contemplate to study the unsteady dusty fluid problem over a continuous Impermeable stretching sheet in the presence of heat source / sink. The highly non-linear, coupled partial differential equations governing the momentum and heat transfer equations are reduced to a system of coupled non-linear ordinary differential equations by applying suitable similarity transformation. These non-linear coupled ordinary differential equations are solved numerically by Runge-Kutta method along with shooting technique for different values of the parameters. These includes the effect of unsteady parameter, heat source/sink parameter, Froud number, Grashof number, Prandtl number, Eckert number, Volume fraction, fluid interaction parameter etc. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived, discussed numerically and their numerical values for various values of physical parameters are presented through tables. A numbers of qualitatively distinct potential scenarios are predicted. It is believed that the results obtained from the present investigation will provide useful information for application and also serve as a complement to the previous investigations.
AMS classification 76T10, 76T15

KEYWORDS: Heat source/sink parameter ,Unsteady parameter, Volume fraction, Fluid – particle interaction parameter,Boundarylayerflow,Stretchingsheet.

INTRODUCTION

In reality, most of the fluids such as molten plastics, polymers, suspension, foods, slurries, paints, glues, printing inks used in the industrial applications are non-Newtonian in nature, especially in polymer processing and chemical engineering processes etc. That is, they might exhibit dramatic deviation from Newtonian behavior depending on the flow configuration and/or the rate of deformation. These fluids often obey nonlinear constitutive equations, and the complexity of their constitutive. The momentum and Heat transfer in the laminar boundary layer flow on a moving surface is important for both practical as well as theoretical point of view because of their wide applications such as ; in heat removal from nuclear fuel debris, the aerodynamic extrusion of plastic sheet, glass blowing, cooling or drying of papers, drawing plastic films, extrusion of polymer melt-spinning process and rolling and manufacturing of plastic films and artificial fibers, waste water treatment, combustion, paint spraying etc. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The mechanical properties of the final product strictly depend on the stretching and cooling rates in the process. The study of boundary layer flow and heat transfer over a stretching sheet has generated much interest in recent years in view of above numerous industrial applications .The study of the boundary layer flow over a stretched surface moving with a constant velocity was initiated by Sakiadis B.C.[4] in 1961. However, according to Wang [23], Sakiadis' solution was not an exact solution of the Navier-Stokes (NS) equations. Then many researchers extended the above study with the effect of heat transfer by considering various aspects of this problem and obtained similarity solutions. A similarity solution is one in which the number of independent variables is reduced by at least one, usually by a coordinate transformation. Grubka et.al [9] investigated the temperature field in the flow over a stretching surface when subject to uniform heat flux. Chen [8] investigated mixed convection of a power law fluid past a stretching surface in presence of thermal radiation and magnetic field .Crane [13] has obtained the Exponential solution for planar viscous flow of linear stretching sheet. B.J. Gireesha et.al [7] have studied the effect of hydrodynamic laminar boundary layer flow and heat Transfer of a dusty fluid over an unsteady stretching surface in presence of non uniform heat source/sink .They have examined the Heat Transfer characteristics for two type of boundary conditions namely variable wall temperature and variable Heat flux. B.J. Gireesha et.al [6] also studied the mixed convective flow a dusty fluid over a stretching sheet in presence of thermal radiation, space dependent heat source/sink. R.N.Barik et.al. [19] have investigated the MHD flow with heat source .Swami Mukhopdhya [20] has studied the Maxwell fluid with heat source and sink. The problem of two phase suspension flow is solved in the frame work of a model of a two-way coupling model or a two-fluid approach. M.S.Uddin [14] et al. has studied MHD Stagnation-Point Flow towards a Heated Stretching Sheet. Anoop Kumar [3] et.al. have studied the Impact of Soret and Sherwood Number on stretching sheet using Homotopy Analysis Method. Tie gang et.al [21] have studied Viscous Flow with Second-Order Slip Velocity over a stretching sheet. P.K. Singh et.al [17] have analyzed MHD flow with viscous dissipation and chemical reaction over a stretching porous plate in porous medium. Noura S. Al-sudais [16] has investigated the thermal radiation effects on MHD fluid flow near stagnation point of linear stretching sheet with variable thermal conductivity. N. Bachok [15] et al.have studied the flow and heat transfer over an unsteady stretching sheet in a micro polar fluid with prescribed surface heat flux. K. V. Prasad [12] et.al have investigated the momentum and heat transfer of a non-Newtonian eyring-powell fluid over a non-isothermal stretching sheet. A.Adhikari [1] et.al have investigated the heat transfer on MHD viscous flow over a stretching sheet with prescribed heat flux. Hitesh Kumar [11] has studied the heat transfer over a stretching porous sheet subjected to power law heat flux in presence of heat source.All of the above mentioned studies dealt with stretching sheet where the unsteady flows of dusty fluid due to a stretching sheet have received less attention; a few of them have considered the two phase flow.

Motivated by the above investigations, in this paper the study of effect of different flow parameters of unsteady boundary layer and heat transfer of a dusty fluid over a stretching sheet have investigated. Here, the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and hence the motion of the particles across the streamline due to the concentration and pressure diffusion. We have considered the terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase, The momentum equation for particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of volume fraction on skin friction, heat transfer and other boundary layer characteristics also have been studied. Further we consider the temperature dependent heat source/sink in the flow. The governing partial differential equations are reduced into system of ordinary differential equations and solved by Shooting Technique using Runge-Kutta Method. To the best of our knowledge this problem has not been considered before, so that the reported results are new.

2. Mathematical formulation and solution:

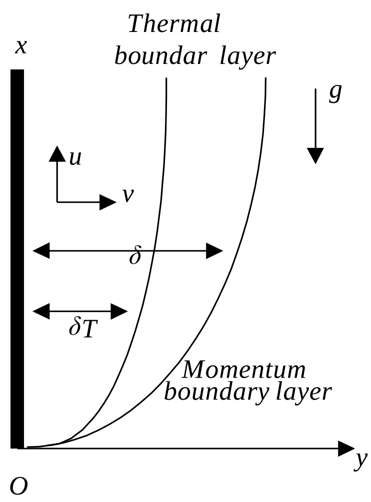

Fig.2.1: Flow analysis & co-ordinate system

Consider an unsteady two dimensional laminar boundary layer of an incompressible viscous dusty fluid over a vertical stretching sheet .The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow .The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperature T_∞ . Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size throughout the flow.

The governing equations of unsteady two dimensional boundary layer incompressible flows of dusty fluids are given by

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$$\frac{\partial u_f}{\partial t}+\frac{\partial}{\partial x}\vec{F}(u_f)+\frac{\partial}{\partial y}\vec{G}(u_f)+H(u_f)$$

$$=S(u_f,u_p,T,T_p) \qquad (2.1)$$

$$\frac{\partial u_p}{\partial t}+\frac{\partial}{\partial x}\vec{F}(u_p)+\frac{\partial}{\partial y}\vec{G}(u_p)+H(u_p)=S_p(u_f,u_p,T,T_p) \qquad (2.2)$$

Where $\vec{F}(u_f)=\begin{bmatrix} u \\ (1-\varphi)\rho u^2 \\ \rho c_p u T\end{bmatrix}$,

$$\vec{G}(u_f)=\begin{bmatrix} v \\ (1-\varphi)\rho uv \\ \rho c_p v T\end{bmatrix} \quad , \quad \vec{F}(u_p)=\begin{bmatrix} \rho_p u_p \\ \rho_p {u_p}^2 \\ \rho_p u_p v_p \\ \rho_p c_s u_p T_p\end{bmatrix} \quad ,$$

$$\vec{G}(u_p)=\begin{bmatrix} \rho_p v_p \\ \rho_p u_p v_p \\ \rho_p {v_p}^2 \\ \rho_p c_s v_p T_p\end{bmatrix}, H(u_f)=0\,, H(u_p)=0$$

$$S(u_f,u_p,T,T_p)=\begin{bmatrix} 0 \\ \mu\frac{\partial^2 u}{\partial y^2}-\frac{\rho_p}{\tau_p}(u-u_p)+g\beta^*(T-T_\infty) \\ k(1-\varphi)\frac{\partial^2 T}{\partial y^2}+\frac{\rho_p c_s}{\tau_T}(T_p-T)+\frac{\rho_p}{\tau_p}(u_p-u)^2+\mu(1-\varphi)\left(\frac{\partial u}{\partial y}\right)^2+(1-\varphi)Q(T-T_\infty)\end{bmatrix}$$

$$S_p(u_f,u_p,T,T_p)=\begin{bmatrix} 0 \\ \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{\rho_p}{\tau_p}(u-u_p)+\varphi(\rho_s-\rho)g \\ \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{\rho_p}{\tau_p}(v-v_p) \\ \frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{\rho_p}{\tau_p}(u-u_p)^2+\varphi\mu_s\left(u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right)-\frac{\rho_p c_s}{\tau_T}(T_p-T)\end{bmatrix}$$

With boundary conditions

$$u=U_w(x,t)=\frac{cx}{1-at}\,, v=0 \; as \; y\to 0$$

$$\rho_p=\omega\rho, u=0\,, u_p=0\,, v_p\to v \; as \; y\to\infty \quad (2.3)$$

Where ω is the density ratio in the main stream.

In order to solve (2.1) and (2.2), we consider non – dimensional temperature boundary conditions as follows

$$T=T_w=T_\infty+T_0\frac{cx^2}{\nu(1-at)^2} \qquad as\; y\to 0$$

$$T\to T_\infty\;, T_p\to T_\infty \quad as\; y\to\infty \qquad (2.4)$$

For most of the gases $\tau_p\approx\tau_T$, $k_s=k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$

if $\frac{c_s}{c_p}=\frac{2}{3P_r}$

Introducing the following non dimensional variables in equation (2.1) and (2.2)

$$u=\frac{cx}{1-at}f'(\eta)\;,\; v=-\sqrt{\frac{c\nu}{1-at}}f(\eta)$$

$$\frac{\varphi\rho_s}{\rho}=\frac{\rho_p}{\rho}=\rho_r=H(\eta)\,, u_p=\frac{cx}{1-at}F(\eta)$$

$$v_p=\sqrt{\frac{c\nu}{1-at}}\;G(\eta)\,,\eta=\sqrt{\frac{c}{\nu(1-at)}}\;y, P_r=\frac{\mu c_p}{k},$$

$$\beta=\frac{1-at}{c\tau_p}\,,\epsilon=\frac{\nu_s}{\nu},\varphi=\frac{\rho_p}{\rho_s}, \mathrm{A}=\frac{a}{c}\,, E_c=\frac{c\nu}{C_pT_0}$$

$$G_r=\frac{g\beta^*(T_w-T_\infty)(1-at)^2}{c^2x}\,, F_r=\frac{c^2x}{g(1-at)^2}\,,\gamma=\frac{\rho_s}{\rho}\,,$$

$$\nu=\frac{\mu}{\rho},\delta=\frac{Qk}{\mu c_p}\frac{R_{ex}}{(R_{ek})^2}\,, R_{ex}=\frac{xU_w}{\nu}\,, R_{ek}=\frac{\sqrt{k}U_w}{\nu},$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\;,\;\theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty} \qquad (2.5)$$

Where

$$T-T_\infty=T_0\frac{cx^2}{\nu(1-at)^2}\theta\,, T_p-T_\infty=T_0\frac{cx^2}{\nu(1-at)^2}\theta_p$$

The equations (2.1) and (2.2) become

$$H'=-(HF+HG')/\left(A\frac{\eta}{2}+G\right) \qquad (2.6)$$

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$$f''' = -ff'' + [f']^2 + A\left[f' + \frac{\eta}{2}f''\right]$$

$$-\frac{1}{(1-\varphi)}\beta\, H[F - f'] - G_r\theta \qquad (2.7)$$

$$A\left[\frac{\eta}{2}F' + F\right] + (F)^2 + GF' - \epsilon F'' + \beta(F - f') - \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) g = 0 \qquad (2.8)$$

$$\frac{A}{2}[\eta G' + G] + GG' = \epsilon G'' - \beta[f + G] \qquad (2.9)$$

$$\theta'' = Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta H}{1-\varphi}(\theta_p - \theta)$$

$$-\frac{PrE_c\beta H}{1-\varphi}(F - f')^2 - PrE_c(f'')^2$$

$$+\frac{A}{2}Pr(\eta\theta' + 4\theta) - Pr\delta\,\theta \qquad (2.10)$$

$$\theta_p'' = \frac{P_r}{\epsilon}\left[\frac{A}{2}(\eta\theta_p' + 4\theta_p) + 2F\,\theta_p + G\theta_p' + \beta(\theta_p - \theta) + \frac{3}{2}E_cP_r\beta(f' - F)^2 - \frac{3}{2}\epsilon E_cP_r(FF'' + (F')^2)\right] \qquad (2.11)$$

With boundary conditions

$$G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1,$$

$$F'(\eta) = 0, \theta(\eta) = 1, \theta_p' = 0 \quad as\ \eta \to 0$$

$$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta), \qquad (2.12)$$

$$H(\eta) = \omega, \theta(\eta) = 0, \theta_p = 0 \quad as\ \eta \to \infty$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which defined as

$C_f = \frac{\tau_w}{\rho U_w^2}$, $Nu_x = \frac{xq_w}{k(T_w - T_\infty)^t}$ where the surface shear stress τ_w and surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using the non dimensional variables, we obtained

$$C_f\, Re_x^{1/2} = f''(0), \quad \frac{Nu_x}{Re_x^{1/2}} = -\theta'(0).$$

3. Solution Method of the problem:

Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ .We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula described in equations (A). Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ *and* $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The essence of shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞)are satisfied.A study was conducting to examine the effect of step size as the appropriate values of step size $\Delta\eta$ was not known to compare the initial values of $F(0), G(0), H(0), \theta_p(0)$, $\theta'(0)$ and $f''(0)$ .If they agreed to about 6 significant digits, the last value of η_∞ used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞ did not lead to any more change in the value of $F(0), G(0), H(0), \theta_p(0)$, $\theta'(0)$ and $f''(0)$ .The step size $\Delta\eta$ =0.125 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}.The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as η =10.0 with a step length of 0.125 beginning from η = 0. Depending upon the initial guess and number of steps N. the value of $f''(0)$ and $\theta'(0)$ are obtained from numerical computation which is given in table – 1 for different parameters.

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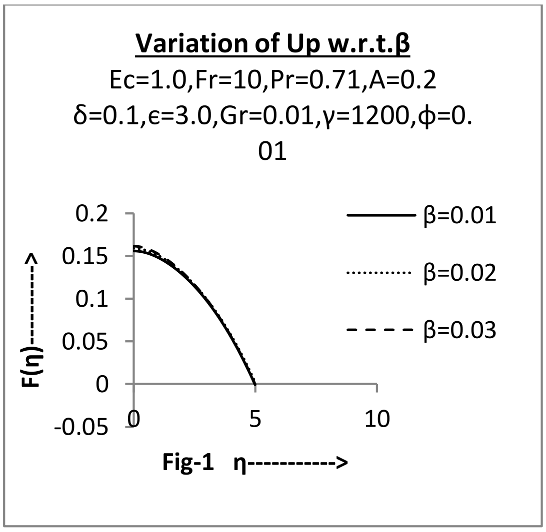
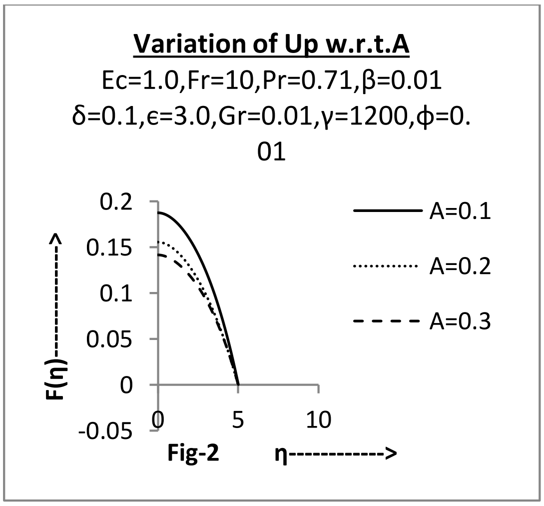
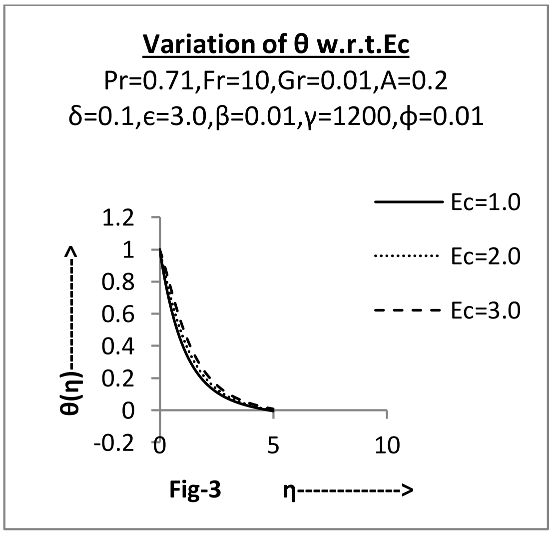
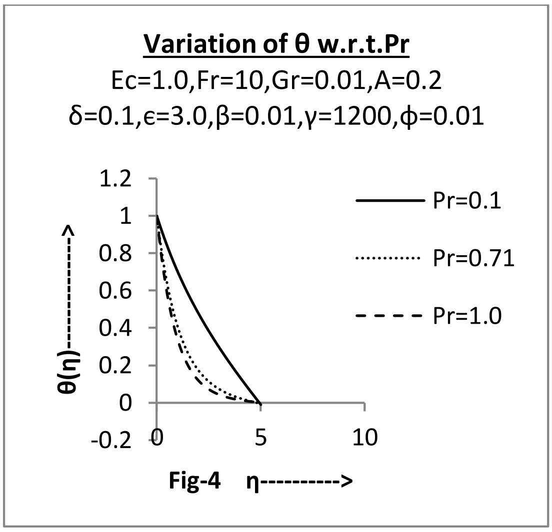
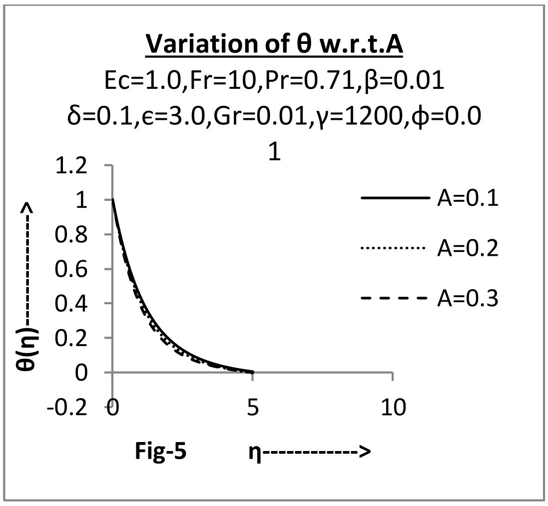
Graphical Representations

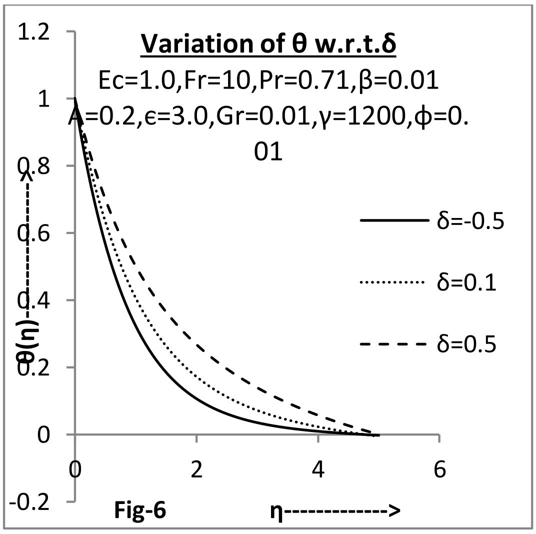












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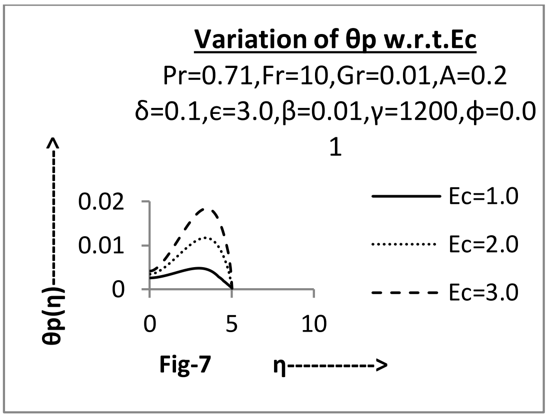

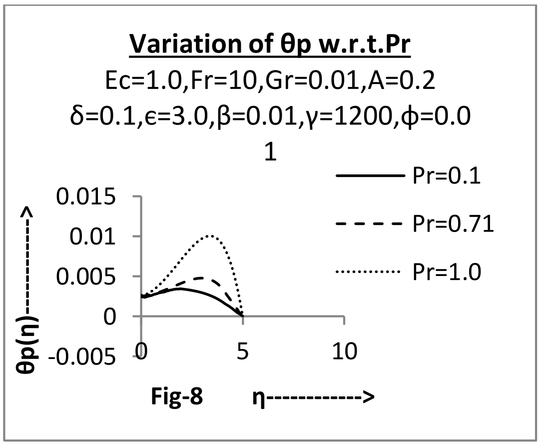

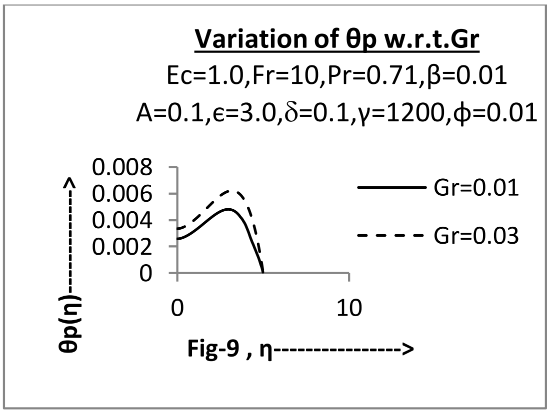

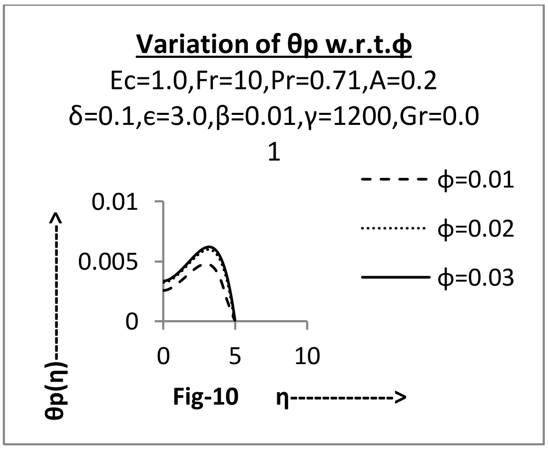

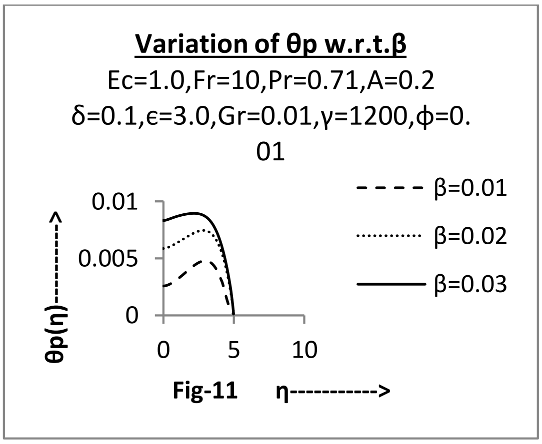

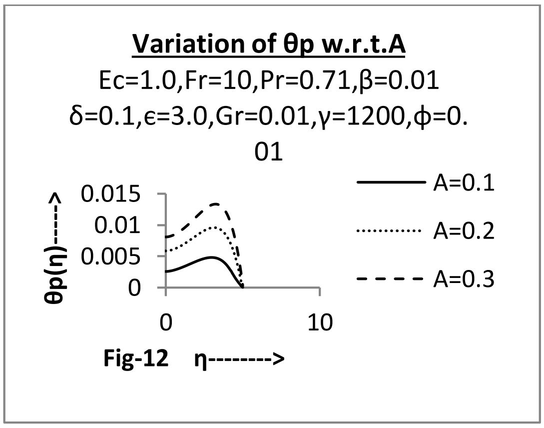

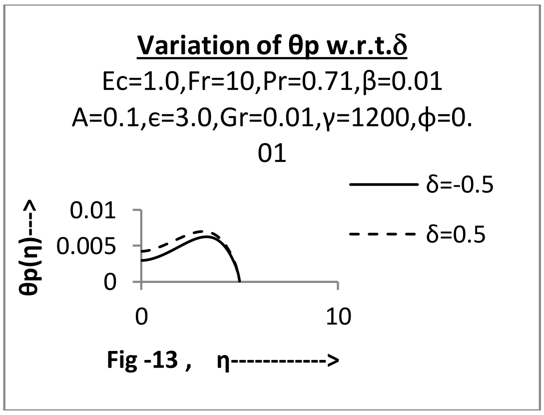

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TABLE-1:
Showing initial values of wall velocity gradient $-f''(0)$ and temperature gradient $-\theta'(0)$

β	δ	A	P_r	E_c	φ	G_r	$-f''(0)$	$u_p(0)$	$-v_p(0)$	H(0)	$-\theta'(0)$	$\theta_p(0)$
0.01	0.1	0.2	0.71	0.00	0.00	0.00	1.079240	-	-	-	1.220351	-
0.01	0.1	0.2	0.71	1.0	0.01	0.01	1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
				2.0			1.06387	0.155974	0.72078	0.111477	0.65741	0.003480
				3.0			1.06316	0.154302	0.72444	0.113379	0.40258	0.004134
			0.1				1.07968	0.147611	0.66129	0.101754	0.60059	0.003714
			0.71				1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
			1.0				1.06502	0.15601	0.72381	0.112103	1.08932	0.002659
						0.01	1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
						0.02	1.05924	0.156032	0.72392	0.111544	0.91468	0.003406
						0.03	1.05391	0.15605	0.72408	0.112407	0.91917	0.003333
					0.01		1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
					0.02		1.0654	0.155788	0.72386	0.112043	0.91127	0.003237
					0.03		1.06445	0.156009	0.72387	0.112045	0.91249	0.00337
		0.2					1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
		0.25					1.08142	0.147379	0.66143	0.103394	0.94028	0.005855
		0.3					1.09796	0.14156	0.06073	0.091406	0.96766	0.008102
0.01							1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
0.02							1.06479	0.159391	0.72175	0.111343	0.91334	0.005865
0.03							1.06520	0.161470	0.72005	0.110756	0.91210	0.008342
	-0.5						1.06509	0.155667	0.72401	0.111083	1.12736	0.002973
	0.1						1.06546	0.155785	0.72388	0.111942	0.91126	0.002570
	0.5						1.06358	0.154876	0.72408	0.112433	0.72180	0.004256

RESULT AND DISCUSSION

The set of non linear ordinary differential equations (2.6) to (2.11) with boundary condition (2.12) were solved using well known Runge-Kutta forth order algorithm with a systematic guessing of $F(0), G(0), H(0), \theta_p(0), f''(0)$ and $\theta'(0)$ by the shooting technique until the boundary condition at infinity are satisfied.The step size 0.125 is used while obtaining the numerical solution accuracy up to the sixth decimal place i.e. 1×10^{-6}, which is very sufficient for convergence. In this method we choose suitable finite values of $\eta \to \infty$ which depends on the values of parameter used. The computations were done by the computer language FORTRAN-77.The shear stress(Skin friction coefficient)which is proportional to $f''(0)$ and rate of heat transfer(Nusselt Number) which is proportional to $\theta'(0)$ are tabulated in Table-1 for different values of parameter used .It is observed from the table that shear stress and rate of heat transfer decreases on the increase of Ec , whereas it is increasing for increasing values of Pr. The Nusselt number decreases on the increasing of unsteady parameter 'A'. The temperature of both phases increase on the increase of δ.

Fig-1 demonstrates the effect of β which infers that increasing of β increases the particle phase velocity. Fig-2 shows that velocity profile of particle phase is decreasing on the increase of unsteady parameter.Fig-3 witnesses that increasing values of Ec, the temperature of fluid phase also increases, it means the frictional heating is responsible for storing heat energy in the fluid.Fig-4 depicts the effect of Pr on temperature profile of fluid phase. From the figure we observe that, when Pr increase the temperature of fluid phase decreases which states that the viscous boundary layer thickness increases and thermal boundary layer thickness decreases.Fig-5 explains that the temperature of fluid phase decreases with increasing unsteady parameter 'A'. This implies it may take less time for cooling during the unsteady flow .Fig-6 explains that the temperature of fluid phase increases with increasing heat generation and decrease on the decrease of heat absorption parameter δ.Fig-7 illustrates the effect of Ec on temperature profile of particle phase. It is evident that the increasing of Ec increases the temperature.Fig-8 explains the effect of Pr on particle phase temperature, when Pr is increasing there is significantly increasing the temperature of particle phase. It means the thermal boundary layer thickness is decreasing.Fig-9 depicts the effect of

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Gr on particle phase temperature profile which indicates that the increasing of Gr has significant effect on particle phase temperature, enhancing Gr, increases the temperature of particle phase.Fig-10 shows the effect of φ on temperature of particle phase. It is evident that increasing in volume fraction increases the temperature of particle phase .Fig-11 illustrates the increasing β, increases the temperature of dust particle.Fig-12 describes that the temperature profile of particle phase increases on the increase of unsteady parameter.Fig-13 describes the effect of δ on particle phase temperature which explains that temperature profile is same as that of fluid phase.

CONCLUSIONS

In this work, it is concluded that, the particle-laden dusty fluid model may predict the following significant conclusions:

i. The increasing value of unsteady parameter A decreases the temperature profiles of fluid phase and increase the dust phase. Also increase value of A decreases velocity of particle phase.
ii. The temperature profile of both phases increases with the increase of heat source /sink parameter δ.
iii. Increasing value of Ec is enhancing the temperature of both fluid phase as well as particle phase which indicates that the heat energy is generated in fluid due to frictional heating.
iv. The thermal boundary layer thickness decreases on the effect of Pr. The temperature decreases at a faster rate for higher values of Pr which implies the rate of cooling is faster in case of higher prandtl number.
v. The momentum boundary layer thickness decreases and thermal boundary layer thickness reduces on the effect of Gr. If Gr =0 the present study will represent the horizontal stretching sheet.
vi. Increasing β increases the particle phase velocity and but temperature profile of particle phase increases.
vii. The effect of φ is negligible on velocity profile but significant effect on temperature profile.
viii. The rate of cooling is much faster for higher values of unsteady parameter but it takes long times for cooling during the steady flow.
ix. Also the local Nusselt number increases with increase of unsteady parameter.
x. We have investigated the problem using the values γ=1200.0 , Fr=10.0 ,ϵ=3.0

Nomenclature:

Q heat source/sink
E_c eckert number

F_r froud number
G_r grashof number

P_r prandtl number

T_∞ temperature at large distance from the wall.

T_p temperature of particle phase.

T_w wall temperature

$U_w(x)$ stretching sheet velocity

c_p specific heat of fluid

c_s specific heat of particles

k_s thermal conductivity of particle

u_p , v_p velocity component of the particle along x-axis and y-axis

A unsteady parameter

c stretching rate

g acceleration due to gravity

k thermal conductivity of fluid

l characteristic length

T temperature of fluid phase.

u,v velocity component of fluid along x-axis and y-axis

x,y cartesian coordinate

Greek Symbols:

δ heat source/sink parameter

φ volume fraction

β fluid – particle interaction parameter

β^* volumetric coefficient of thermal expansion

ρ density of the fluid

ρ_p density of the particle phase

ρ_s material density

η similarity variable

θ fluid phase temperature

θ_p dust phase temperature

μ dynamic viscosity of fluid

ν kinematic viscosity of fluid

γ ratio of specific heat

τ relaxation time of particle phase

τ_T thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.

τ_p velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.

ε diffusion parameter

ω density ratio

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